Summer Math Packet for Students Entering Algebra I

Hello Students!

Welcome to summer.

As you prepare for Algebra, it is important to review some skills foundational skills.

This math packet is a compilation of exercises that include explanations.

Please follow these directions

- Please put your name on each page.
- Please do not use a calculator if the page states "No Calculator".
- There are hints on the top of each page. Read them before beginning the work.
- Follow the directions on each page.
- You must show your work for full credit.
- This packet will be graded. It is due on the first day of school. It will count as a quiz grade.
- Simplify answers when possible.

Here is a suggested schedule:

Week1: Integers – Adding and Subtracting (page 1)

Week 2: Integers – Multiplying and Dividing (page 2)

Week 3: Greatest Common Factor and Least Common Multiple (pages 3 & 4)

Week 4: Fractions: Adding and Subtracting (page 5)

Week 5: Fractions: Multiplying and Dividing (page 6)

Week 6: Order of Operations; Expressions (pages 7 & 8)

Week 7: Solving Equations (pages 9 & 10)

Week 8: Inequalities (page 11)

Week 9: Multiplying Monomials (page 12 & 13))

Week 10: Distributive Property (page 14)

Week 11: Combining Like Terms (pages 15 & 16)

I hope you have a wonderful time this summer. It will be fun to see you next fall and hear about your adventures.

Integers – Adding and Subtracting

Rules: ** If a number has no sign it means it is a positive number. **

Addition

SAME SIGNS

1) Add their absolute values.
2) Attach the common signs.

-4 + (- 5) = -(4 + 5) = -9 4 + 5 = 9

OPPOSITE SIGNS

1) Subtract the smaller absolute value from the larger absolute value.
2) Attach the sign of the number with the larger absolute value.

3 + (-9) = -(9 – 3) = -6 -3 + 9 = +(9 – 3) = 6

Subtraction

1) Adding the opposite of a number is equivalent to subtracting the number.
2) Change all problems to addition and follow the addition rules.

3 – 12 = 3 + (-12) = -(12 – 3) = -9

-7 – 1 = -7 + (-1) = -(7 + 1) = -8

-4 – (-10) = -4 + 10 = +(10 – 4) = 6

12 – (-8) = 12 + 8 = 20

NO CALCULATOR!

1. 7 + (-9) =	2. – 12 + 15 =
3. 2 – 4 =	4. 12 – 19 =
5. -7 – (-5) =	6. 7 + 27 =
7. – 12 – (-4) =	8. 0 – 8 =
9. 0 – (-6) =	10. -8 – 2 =
11. – 3 + 1 =	12. -7 + (-5) =
13. -9 – (-13) + (-4)=	14. -6 – 4 – (-8) =
15. 25 – 21 + (-20) =	16. -39 – (-32) – 14 =

Integers – Multiplying and Dividing

Rules:

1) If two numbers have the same sign, their product or quotient is positive.
   (-7)(-5) = 35 $6 \bullet 8 = 48$
2) If two numbers have opposite signs, their product or quotient is negative
   9(-2) = -18 (-3)(4) = -12

NO CALCULATOR!

1. (-8)(3) =	2. (4)(-4) =
3. (20)(-65) =	4. $-7 \bullet -5$ =
5. -45 ÷ 9 =	6. $\frac{-24}{-4}$ =
7. 49 ÷ (-7) =	8. $\frac{-99}{9}$ =
9. (5)(-2)(7) =	10. (-3)(-1)(4)(-6) =
11. -3740 ÷ (-10) =	12. $\frac{56}{-7}$ =
13. (11)(-1)(-7)(-3) =	14. $\frac{39}{13}$ =
15. (-72) ÷ (-12) =	16. (-9)(8)(-2)(5) =

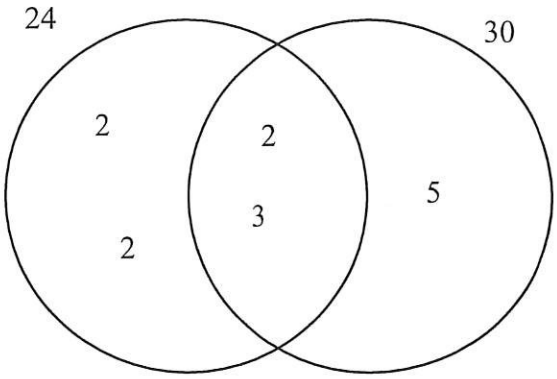
Notes

Greatest Common Factor in Venn Diagrams

Example: Find the greatest common factor of 24 and 30, GCF(24,30)
Prime factorizations:

$24 = 2^3 \times 3$

$30 = 2 \times 3 \times 5$

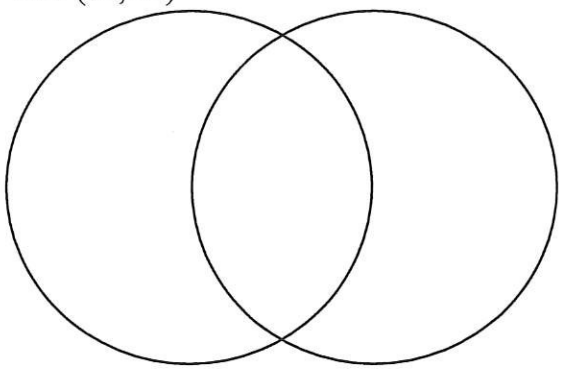


The prime factorizations of 24 and 30 have two numbers in common; one 2 and one 3. These go in the intersection of the two circles on the Venn diagram. The "other" factors are placed as shown on the right. **The greatest common factor will be the product of the numbers in the intersection.**

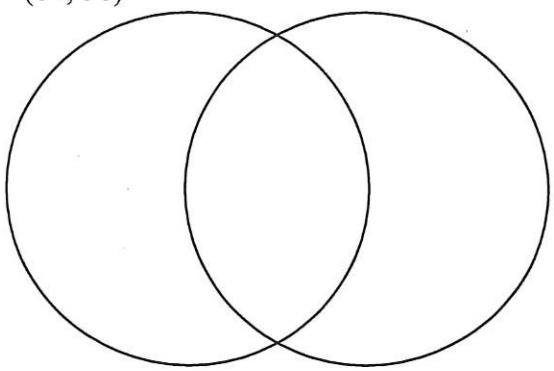
By looking at the Venn Diagram, we can see that the GCF(24,30) $= 2 \times 3 = 6$.

Practice Problems: find the prime factorization of each number. Then place the prime factors in the appropriate part of the Venn diagram and find the GCF of the numbers.

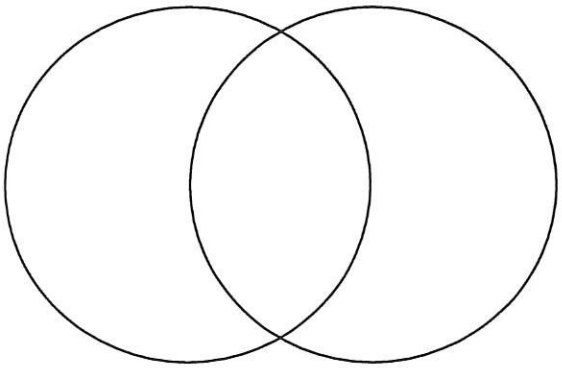
GCF(72, 54) =



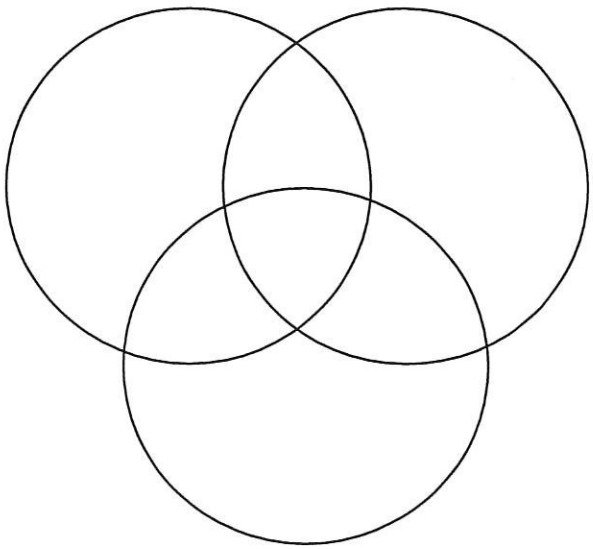
GCF(84, 56) =



GCF(21,10) =



GCF(18,45,60) =



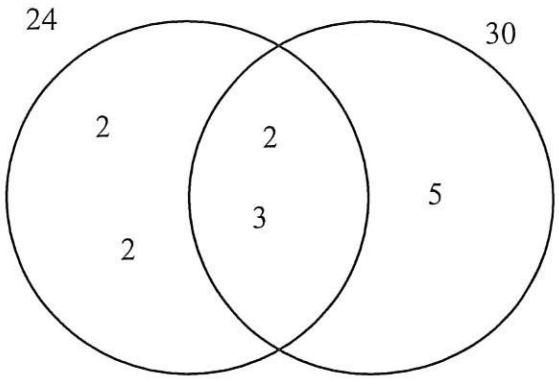
Notes

Least Common Multiple in Venn Diagrams

Example: Find the least common multiple of 24 and 30, LCM(24,30)
Prime factorizations:

$24 = 2^3 \times 3$

$30 = 2 \times 3 \times 5$

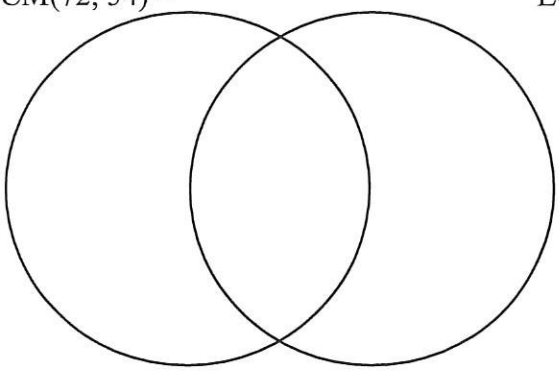


The prime factorizations of 24 and 30 have two numbers in common; one 2 and one 3. These go in the intersection of the two circles on the Venn diagram. The "other" factors are placed as shown on the right. **The least common multiple will be will be the product of the numbers in the Venn diagram.**

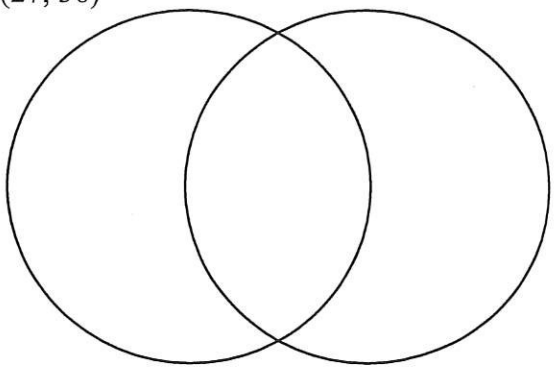
By looking at the Venn Diagram, we can see that the LCM(24,30) $= 2 \times 2 \times 2 \times 3 \times 5 = 120$.

Practice Problems: find the prime factorization of each number. Then place the prime factors in the appropriate part of the Venn diagram and find the LCM of the numbers.

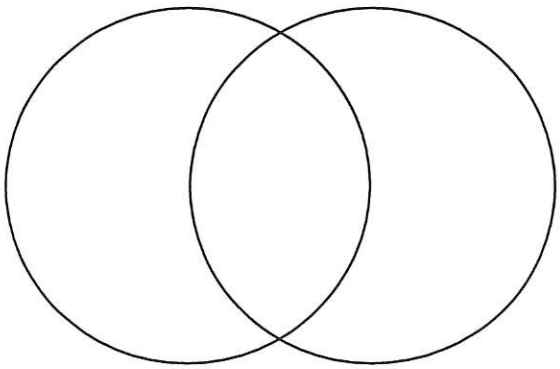
LCM(72, 54) =



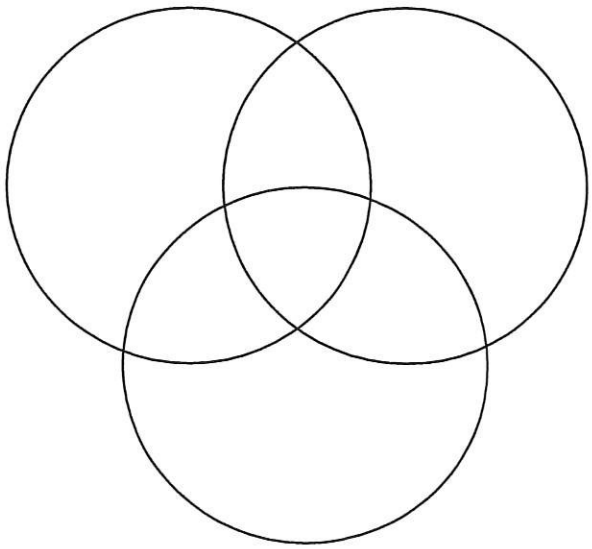
LCM(27, 36) =



LCM(9,16) =



LCM(24,90,100) =



Fractions – Adding and Subtracting

Rules:

1) Find LCD.
2) Change to equivalent fractions.
3) Rename, if needed.
4) Add or Subtract.
5) Simplify

$3\frac{1}{9} = 3\frac{2}{18} = 2\frac{20}{18}$

$-1\frac{5}{6} = -1\frac{15}{18} = -1\frac{15}{18}$

$1\frac{5}{18}$

$4\frac{3}{4} = 4\frac{9}{12}$

$+ \quad 5\frac{5}{6} = +5\frac{10}{12}$

$9\frac{19}{12} = 10\frac{7}{12}$

NO CALCULATOR!

1. $2\frac{3}{4}+5\frac{5}{6}$	2. $9-4\frac{2}{5}$	3. $6\frac{1}{3}+4\frac{3}{5}$	4. $8\frac{1}{9}-2\frac{5}{6}$
5. $9+1\frac{1}{7}$	6. $6\frac{1}{2}+2\frac{2}{3}$	7. $5\frac{1}{2}+1\frac{3}{5}$	8. $1\frac{3}{4}-\frac{1}{2}$
9. $\frac{1}{5}+1\frac{3}{4}$	10. $\frac{4}{5}-\frac{2}{3}$	11. $\frac{5}{7}+1\frac{4}{5}$	12. $3\frac{5}{8}-2\frac{1}{6}$

Fractions – Multiplying and Dividing

Rules:

1) Change all mixed numbers to improper fractions.
2) Multiplying across.
3) Simplify

$$2\frac{2}{3}\cdot 4\frac{1}{10}=\frac{8}{3}\cdot\frac{41}{10}=\frac{4}{3}\cdot\frac{41}{5}=\frac{164}{15}=10\frac{14}{15}$$

1) Change all mixed numbers to improper fractions.
2) Take the reciprocal.
3) Multiply across.
4) Simplify

$$2\frac{3}{4}\div 3\frac{1}{2}=\frac{11}{4}\div\frac{7}{2}=\frac{11}{4}\cdot-\frac{2}{7}=\frac{11}{2}\cdot\frac{1}{7}=\frac{11}{14}$$

NO CALCULATOR!

1. $2\frac{3}{4}\bullet 1\frac{5}{11}$	2. $9\bullet 4\frac{2}{3}$	3. $1\frac{1}{3}\bullet 4\frac{1}{6}$	4. $1\frac{1}{9}\bullet 2\frac{2}{5}$
5. $9\bullet 1\frac{1}{3}$	6. $6\frac{1}{2}\bullet 2\frac{1}{13}$	7. $5\frac{1}{2}\div 1\frac{3}{4}$	8. $1\frac{3}{4}\div\frac{1}{2}$
9. $\frac{1}{5}\div 1\frac{3}{4}$	10. $\frac{4}{5}\div\frac{2}{3}$	11. $\frac{9}{20}\div 1\frac{4}{5}$	12. $3\frac{2}{8}\div 2\frac{1}{6}$

Order of Operations

Parentheses (Grouping Symbols)	$[(7-4)^2+3]+15$	$\frac{(9-7)^2+6}{11-6}$
Exponents	$=[3^2+3]+15$	$=\frac{2^2+6}{5}$
Multiply or Divide, from left to right	$=[9+3]+15$	$=\frac{4+6}{5}$
Add or Subtract, from left to right	$=12+15$	$=\frac{10}{2}$
		$=5$

NO CALCULATOR!

1. $6 \div 3 + 2 \bullet 7$	2. $5 + 8 \bullet 2 - 4$	3. $16 \div 8 \bullet 2^2$	4. $10 \div (3 + 2) + 9$
5. $7[(18-6)-6]$	6. $3(2.7 \div 0.9) - 5$	7. $6(5-3)^2 + 3$	8. $[10 + (5^2 \bullet 2)] \div 6$
9. $\frac{1}{3}(9 \bullet 3) + 18$	10. $\frac{1}{2} \bullet 26 - 3^2$	11. $2.5 \bullet 0.5^2 \div 5$	12. $\frac{16}{8} + 2^3 - 10$
13. $\frac{9 \cdot 2}{4+3^2-1}$	14. $\frac{13-4}{18-4^2+1}$	15. $\frac{5^3 \bullet 2}{1+6^2-8}$	16. $\frac{7 \bullet 4}{8+7^2-1}$

Expressions

Write the verbal phrase as an algebraic expression. Eleven less than the quantity four times a number x	$4(x-11)$
Evaluate the expression x^2+4-x, when $x=6$	6^2+4-6 $=36+4-6$ $=40-6$ $=34$

Write the verbal phrase as an algebraic expression.

1. four times a number x decreased by twelve	2. six less than double a number x
3. five squared minus a number x	4. three more than the product of five and number x
5. twenty-nine decreased by triple a number x	6. two cubed divided by a number x
7. the quotient of a number x and two-tenths	8. the difference of ten and a number x

NO CALCULATOR!

Evaluate the expression

9. $y \div 3+2$, when $y=30$	10. $\frac{r}{s}\cdot 7$, when $r=30$ and $s=5$
11. $5x^2-y$, when $x=4$ and $y=26$	12. $3r^2-17$, when $r=6$
13. $\frac{4}{5}\div n+13$, when $n=\frac{1}{5}$	14. $\frac{9}{10}\bullet y-\frac{3}{10}$, when $y=\frac{1}{2}$

Solving Equations I

Hints/Guide:

The key in equation solving is to isolate the variable, to get the letter by itself. In one-step equations, we merely undo the operation - addition is the opposite of subtraction and multiplication is the opposite of division. Remember the golden rule of equation solving: If we do something to one side of the equation, we must do the exact same thing to the other side. Examples:

1. $x + 5 = 6$
   $-5 \quad -5$
   $x = 1$
   Check: $1 + 5 = 6$
   $6 = 6$

2. $t - 6 = 7$
   $+6 \quad +6$
   $t = 13$
   Check: $13 - 6 = 7$
   $7 = 7$

3. $\frac{4x}{4} = \frac{16}{4}$
   $x = 4$
   Check: $4(4) = 16$
   $16 = 16$

4. $6 \cdot \frac{r}{6} = 12 \cdot 6$
   $r = 72$
   Check: $72 \div 6 = 12$
   $12 = 12$

Exercises: Solve the following problems: No Calculators!
SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $x + 8 = -13$
2. $t - (-9) = 4$
3. $-4t = -12$
4. $\frac{r}{4} = 24$
5. $y - 4 = -3$
6. $h + 8 = -5$
7. $\frac{p}{8} = -16$
8. $-5k = 20$
9. $-9 - p = 17$

Equations - Variables on Each Side

Hints/Guide:

As we know, the key in equation solving is to isolate the variable. In equations with variables on each side of the equation, we must combine the variables first by adding or subtracting the amount of one variable on each side of the equation to have a variable term on one side of the equation. Then, we must undo the addition and subtraction, then multiplication and division. Remember the golden rule of equation solving. Examples:

$$\begin{aligned} 8x - 6 &= 4x + 5 \\ -4x \quad & \quad -4x \\ 4x - 6 &= \quad 5 \\ +6 \quad & \quad +6 \\ \frac{4x}{4} &= \frac{11}{4} \\ x &= 2\frac{3}{4} \end{aligned}$$

$$\begin{aligned} 5 - 6t &= 24 + 4t \\ +6t \quad & \quad +6t \\ 5 \quad & = 24 + 10t \\ -24 \quad & \quad -24 \\ \frac{-19}{10} &= \frac{10t}{10} \\ -1\frac{9}{10} &= t \end{aligned}$$

Exercises: Solve the following problems: No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $4r - 7 = 8r + 13$
2. $14 + 3t = 5t - 12$
3. $4x + 5 = 3x - 3$
4. $6y + 5 = 4y - 13$
5. $5x - 8 = 6 - 2x$
6. $7p - 8 = -4p + 6$

Inequalities

Hints/Guide:

In solving inequalities, the solution process is very similar to solving equalities. The goal is still to isolate the variable, to get the letter by itself. However, the one difference between equations and inequalities is that when solving inequalities, when we multiply or divide by a negative number, we must change the direction of the inequality. Also, since an inequality as many solutions, we can represent the solution of an inequality by a set of numbers or by the numbers on a number line.

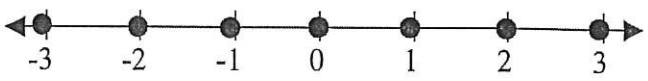
Inequality - a statement containing one of the following symbols:

$<$ is less than $>$ is greater than $\leq$ is less than or equal to

$\geq$ is greater than or equal to _ is not equal to

Examples:

1. Integers between -4 and 4.

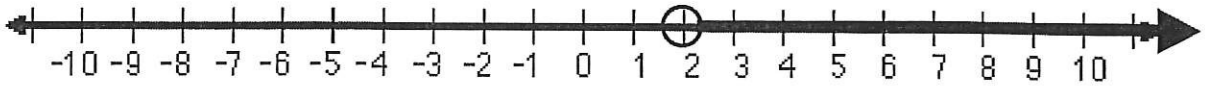


2. All numbers between -4 and 4.

3. The positive numbers.

So, to solve the inequality $-4x < -8$ becomes $\frac{-4x}{-4} < \frac{-8}{-4}$

and therefore $x > 2$ is the solution (this is because whenever we multiply or divide an inequality by a negative number, the direction of the inequality must change) and can be represented as:



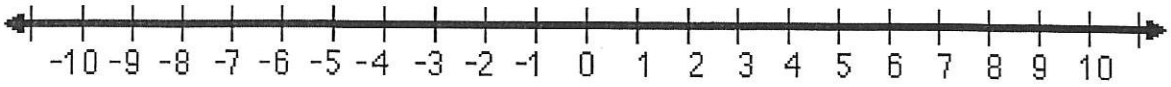
Exercises: Solve the following problems: No Calculators!

1. $4x > 9$

2. $-5t \geq -15$

3. $\frac{x}{2} \geq 3$

4. $\frac{x}{-4} > 2$



Name _____

Multiplying Monomials Lesson

Multiply: $(4xy^2)\ (2xy^3)$

Step 1) Rearrange the terms.

Everything is being multiplied. Because multiplication is commutative, the order doesn't matter. We will start by rearranging the terms in such a way that they are easier to work with.

$$(4xy^2)\ (2xy^3) = \underbrace{4 \bullet 2}_{\text{numbers}} \bullet \underbrace{x \bullet x}_{\text{x terms}} \bullet \underbrace{y^2 \bullet y^3}_{\text{remaining y terms}}$$

Step 2) Combine the pieces by multiplying them.

$$\underbrace{4 \bullet 2}_{8} \bullet \underbrace{x \bullet x}_{x^2} \bullet \underbrace{y^2 \bullet y^3}_{y^5}$$

Final answer: $8x^2y^5$

Name _____

Multiplying Monomials Worksheet 3

Find each product.

1) $6g^2 \bullet 5g =$ _____

2) $-8h^3 \bullet 4h^2 =$ _____

3) $-10v^3 \bullet 10v =$ _____

4) $-4x^5 \bullet 5x^3 =$ _____

5) $7b^3 \bullet 9b =$ _____

6) $2m^4 \bullet 10m^2 =$ _____

7) $-9n^2 \bullet 7n^3 =$ _____

8) $5y^5 \bullet 5y =$ _____

9) $-8z^2 \bullet -4z^3 =$ _____

10) $3p \bullet 6p =$ _____

11) $9k^3 \bullet 6k =$ _____

12) $6c^7 \bullet 5c =$ _____

Distributive Property

Distributive Property	
$a(c) = ab + bc$	$3(2x + 1) = 6x + 3$
$(b + c)a = ba + ca$	$(4x + 5)x = 4x^2 + 5x$
$a(b - c) = ab - ac$	$-9(x - 8) = -9x + 72$
$(b - c)a = ba - ca$	$(x^2 - 3)x = x^3 - 3x$

Distribute

1. $3(x + 4)$	2. $(w + 6)4$	3. $5(y - 2)$	4. $(7 - m)8$
5. $-(y - 9)$	6. $(-2)(x + 6)$	7. $(2x - 4)(-3)$	8. $x(x + 1)$
9. $-9(a + 6)$	10. $4x(x + 8)$	11. $-2t(12 - t)$	12. $(3y - 2)5y$
13. $-2x(x - 8)$	14. $-9(-t - 3)$	15. $(6 - 3w)(-w^2)$	16. $-y(-y^2 + y)$

Name _____ Date _____

Notes

Combining Like Terms

One way to simplify an expression is to "combine like terms."

What does it mean to combine like terms?	

You can only combine terms that have the same _____ *and the same*

_____.

To combine like terms, ***first*** *use the commutative property to move all like terms together.* ***Then****, combine the coefficients of the variables.*

Example 1:

$2a + 3b - 4a$

$2a - 4a + 3b$

$-2a + 3b$

Note: Make sure to move any negative signs with the term it is before!

Example 1:

$14m - 3n^2 - 2n^2 + 3m$

Example 1:

$5x + 4x - 6 + 5x^2$

Note: all of your answers should be arranged so that the variables are in _____

order first, then in order from greatest to least _____.

Watch out for the following common mistakes! Circle the mistakes below:

Mistake #1:

$a^2 - 4a + 5a$

$2a^2$

You can ***ONLY*** *combine terms when the variable has the same exponent.*

Mistake #2:

$3y + 4x^2 - 3y + 5y$

$3y - y + 5y + 4x^2$

$7y + 4x^2$

You should ***ALWAYS*** *put the variables of your answer in alphabetical order, then in order by exponent.*

Mistake #3:

$3h + 14g - 5h + 5g$

$3h + 5h + 14g - 5g$

$8h + 9g$

$9g + 8h$

You should ***ALWAYS*** *move the negative sign along with the term that is after it.*

Name _____ Date _____

Practice

Combining Like Terms Puzzle

Simplify each expression by combining like terms. Find the answer at the bottom of the page. Then write the letter on the appropriate line below to spell out a secret message. (Some letters may be used more than once!)

Did you hear the one about the acupuncture?

___	___		___	___	___		___		___	___	___		___	___	___	___	!
1	2		3	4	5		6		7	8	9		10	11	12	13	
													14	15	16	17	

1. $2m + 3m^2 - 4m$	2. $2x + x - 4y$	3. $2m + 4m - 3m^2$	4. $2y + 14x - 7x + 9y$
5. $8n - 4n^2 + 8n$	6. $11g - 9g + 8g$	7. $3m^2 - 2m + 4m$	8. $20 + 10q + 3q - 4$
9. $4xy + x + 2xy$	10. $6m^2 + 6m - 9m^2$	11. $3n - 6mn + 2n$	12. $\frac{3}{2}x - y + \frac{1}{2}x + 3y$
13. $y + x + y + x$	14. $8n + 4n^2 - 8n$	15. $5 + 5mn - 11mn$	16. $15y + 6y - 3x + xy$
17. $3xy - 5xy + 21y$			

I. $3m^2 - 2m$	**N. $-3x + xy + 21y$**	**A. $7x + 11y$**
S. $-4n^2 + 16n$	**W. $-3m^2 + 6m$**	**E. $-6mn + 5n$**
E. $-2xy + 21y$	**J. $3m^2 + 2m$**	**A. $13q + 16$**
O. $-6mn + 5$	**B. $x + 6xy$**	**L. $2x + 2y$**
T. $3x - 4y$	**A. $10g$**	**D. $4n^2$**